

The Collatz Conjecture

Unit: Number Play - Student Practice Worksheet

Overview: Ms. Rao gave a simple rule: if a number is even, halve it; if it's odd, triple it and add 1. Repeat forever. Kabir started with 6 and reached 1 in eight steps. Anaya started with 27 and kept going... and going... for over a hundred steps. No matter which starting number anyone has ever tried, this strange bouncing sequence has always eventually landed on 1 — but nobody has ever proven it always must.

Practice Questions (30 Total)

Section A - Easy

Q1. The teal card shows the even rule. What do you do to an even number?

$$\text{even} \rightarrow n / 2$$

$$6 \rightarrow 3$$

$$\text{odd} \rightarrow 3n + 1$$

$$3 \rightarrow 10$$

- ☐ A. Divide it by 2
- ☐ B. Multiply it by 3
- ☐ C. Add 1 to it
- ☐ D. Subtract 1

Q2. What is a conjecture?

- ☐ A. A believed but unproven mathematical statement
- ☐ B. A formally proven fact
- ☐ C. A type of equation
- ☐ D. A calculator function

Q3. Starting from 6, what is the next number in the Collatz sequence?

- ☐ A. 12
- ☐ B. 19
- ☐ C. 3
- ☐ D. 18

Q4. The peach card shows the odd rule on 3. What is the next number?

$$3 \times 3 + 1 = 10$$

- ☐ A. 1.5
- ☐ B. 6
- ☐ C. 9
- ☐ D. 10

Q5. In the Collatz rule, what do you do to an odd number?

- ☐ A. Divide it by 2
- ☐ B. Multiply by 3 and add 1
- ☐ C. Subtract 3
- ☐ D. Multiply by 2

Q6. Starting from 10, what is the next number?

- ☐ A. 20
- ☐ B. 31
- ☐ C. 9
- ☐ D. 5

Q7. The $\div 2$ arrow halves 10. What lands in the peach box?



- ☐ A. 20
- ☐ B. 5
- ☐ C. 11
- ☐ D. 9

Q8. Is the Collatz Conjecture formally proven for ALL numbers?

- ☐ A. Yes, it was proven long ago
- ☐ B. It was disproven
- ☐ C. No, it remains unproven

- ☐ D. It only applies to even numbers

Q9. Starting from 5, what is the next number?

- ☐ A. 16
- ☐ B. 2.5
- ☐ C. 10
- ☐ D. 15

Q10. What number does every tested Collatz sequence eventually reach?

- ☐ A. 0
- ☐ B. 1
- ☐ C. 10
- ☐ D. 100

Section B - Medium

Q11. Count the arrows from 6 to 1 on this path. How many steps is that?

$6 \rightarrow 3 \rightarrow 10 \rightarrow 5 \rightarrow 16$
 $\rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$

- ☐ A. 5
- ☐ B. 8
- ☐ C. 12
- ☐ D. 3

Q12. What makes the Collatz Conjecture surprising to mathematicians?

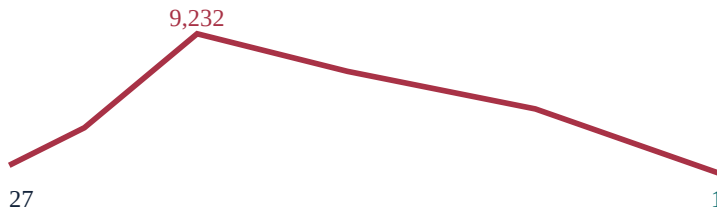
- ☐ A. Its rule is very simple, yet nobody can prove it always works
- ☐ B. It has been fully proven and is not surprising at all
- ☐ C. It only applies to prime numbers
- ☐ D. It was solved instantly when proposed

Q13. Starting from 9, the sequence begins 9, 28, 14. What is the next number?

- ☐ A. 7
- ☐ B. 42

- ☐ C. 15
- ☐ D. 28

Q14. The sketch of 27's ride peaks at the labelled height. How high does it climb?



- ☐ A. 100
- ☐ B. 1,000
- ☐ C. 9,232
- ☐ D. 27

Q15. What is the difference between a conjecture and a proven theorem?

- ☐ A. A conjecture lacks a formal proof; a theorem has one
- ☐ B. They mean exactly the same thing
- ☐ C. A theorem is always false
- ☐ D. A conjecture only applies to even numbers

Q16. How many steps did Anaya's sequence starting from 27 take to reach 1?

- ☐ A. 27
- ☐ B. 50
- ☐ C. 9
- ☐ D. 111

Q17. This path stays even the whole way. Why do powers of 2 reach 1 quickly?

$$16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

- ☐ A. They are actually the slowest numbers
- ☐ B. Powers of 2 never reach 1
- ☐ C. They stay even every step, just halving with no odd detours
- ☐ D. They skip straight to 1 immediately

Q18. How has the Collatz Conjecture been tested by mathematicians?

- ☐ A. By hand, checking exactly 100 numbers
- ☐ B. It has never been tested at all
- ☐ C. Only for numbers under 10
- ☐ D. By computer, checking trillions of starting numbers

Q19. Starting from 20, what is the very next step?

- ☐ A. 61
- ☐ B. 10
- ☐ C. 40
- ☐ D. 19

Q20. Does the Collatz sequence starting from any tested number eventually reach 1?

- ☐ A. No, most numbers never reach 1
- ☐ B. Yes, in every tested case so far
- ☐ C. Only even starting numbers reach 1
- ☐ D. Only odd starting numbers reach 1

Section C - Challenge

Q21. The three circles show what happens after 1. What loop is that?



- ☐ A. It loops forever as 1,4,2,1,4,2...
- ☐ B. The sequence stops with no more steps
- ☐ C. 1 becomes 0
- ☐ D. 1 is treated as even

Q22. Why isn't 'it worked for trillions of tested numbers' considered a mathematical proof?

- ☐ A. A proof must show it works for every possible case, not just tested ones
- ☐ B. It actually IS a full proof
- ☐ C. Trillions is not a large enough number to matter

- ☐ **D.** Computers cannot be trusted with arithmetic

Q23. Starting from 16, list the sequence down to 1.

- ☐ **A.** 16,32,64,128
- ☐ **B.** 16,8,4,2,1
- ☐ **C.** 16,15,14,13
- ☐ **D.** 16,4,1

Q24. The two lines explain $3n+1$ after an odd n . Why is that result always even?

odd $\times 3 \rightarrow$ still odd

odd $+ 1 \rightarrow$ even, so next step halves

- ☐ **A.** It's actually random whether the result is even or odd
- ☐ **B.** $3n+1$ always produces an odd result
- ☐ **C.** This is only true for numbers under 100
- ☐ **D.** 3 times odd is odd, and odd plus 1 is even

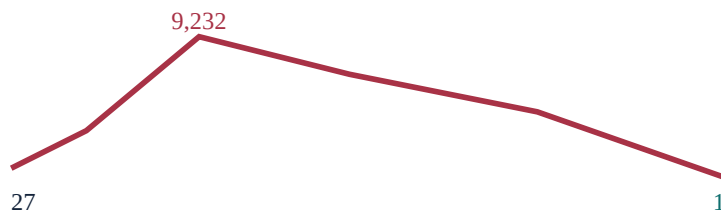
Q25. Why might a small starting number like 27 produce a much longer sequence than a bigger number like 1,024?

- ☐ **A.** Bigger numbers always take longer, no exceptions
- ☐ **B.** The path length depends on the odd/even pattern, not just starting size
- ☐ **C.** 27 is actually not a valid Collatz starting number
- ☐ **D.** Smaller numbers always take fewer steps

Q26. From 9, after $9 \rightarrow 28 \rightarrow 14 \rightarrow 7$, what is the next number?

- ☐ **A.** 22
- ☐ **B.** 3.5
- ☐ **C.** 8
- ☐ **D.** 21

Q27. 27's path peaks then falls. How many steps does that whole ride take to reach 1?



- ☐ A. 8
- ☐ B. 27
- ☐ C. 111
- ☐ D. 9,232

Q28. What would a single counterexample (a starting number that never reaches 1) do to the conjecture?

- ☐ A. It would have no effect at all
- ☐ B. It would make the conjecture more likely true
- ☐ C. It would disprove the conjecture completely
- ☐ D. It would only affect even numbers

Q29. If mathematicians prove the Collatz Conjecture true, what would that formally establish?

- ☐ A. That only even numbers reach 1
- ☐ B. That the rule needs to be changed
- ☐ C. That no number ever reaches 1
- ☐ D. That EVERY whole number eventually reaches 1

Q30. Kabir wonders if a starting number could loop forever without reaching 1. What does current evidence suggest?

- ☐ A. Loops are common and expected
- ☐ B. It has been proven impossible
- ☐ C. None found, but not proven impossible
- ☐ D. Only odd numbers can loop

Answer Key

- Q1:** (A) (a) Divide it by 2. The teal card is $n / 2$, as in $6 \rightarrow 3$.
- Q2:** (A) (a) A believed but unproven mathematical statement.
- Q3:** (C) (c) 3. 6 is even, so $6/2=3$.
- Q4:** (D) (d) 10. 3 is odd, so $3 \times 3 + 1 = 10$.
- Q5:** (B) (b) Multiply by 3 and add 1.
- Q6:** (D) (d) 5. 10 is even, so $10/2=5$.
- Q7:** (B) (b) 5. 10 is even, so $10/2=5$.
- Q8:** (C) (c) No, it remains unproven despite extensive testing.
- Q9:** (A) (a) 16. 5 is odd, so $5 \times 3 + 1 = 16$.
- Q10:** (B) (b) 1.
- Q11:** (B) (b) 8. $6 \rightarrow 3 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$ is eight steps. The hook once said five by mistake.
- Q12:** (A) (a) Its rule is very simple, yet nobody can prove it always works.
- Q13:** (A) (a) 7. 14 is even, so $14/2=7$.
- Q14:** (C) (c) 9,232 — the labelled peak on the path from 27 down to 1.
- Q15:** (A) (a) A conjecture lacks a formal proof; a theorem has one.
- Q16:** (D) (d) 111 steps — far longer than the small starting number would suggest.
- Q17:** (C) (c) They stay even every step, just halving repeatedly with no odd detours.
- Q18:** (D) (d) By computer, checking trillions of starting numbers.
- Q19:** (B) (b) 10. 20 is even, so $20/2=10$.
- Q20:** (B) (b) Yes, in every tested case so far.
- Q21:** (A) (a) 1 is odd, so $1 \rightarrow 4 \rightarrow 2 \rightarrow 1$, looping forever — that is why 1 is the stopping point.
- Q22:** (A) (a) There are infinitely many whole numbers, so testing alone cannot cover them all.
- Q23:** (B) (b) 16,8,4,2,1 — repeatedly halving a power of 2.
- Q24:** (D) (d) 3 times an odd number is odd, and adding 1 makes it even, so the next step always halves.
- Q25:** (B) (b) 1,024 is a pure power of 2, so it halves all the way down very quickly.
- Q26:** (A) (a) 22. 7 is odd, so $7 \times 3 + 1 = 22$.
- Q27:** (C) (c) 111 steps, even though the sketch's peak is 9,232.
- Q28:** (C) (c) It would disprove the conjecture completely — one genuine exception is enough.
- Q29:** (D) (d) That EVERY whole number, without exception, eventually reaches 1.
- Q30:** (C) (c) No such loop has ever been found, but it hasn't been proven impossible either.
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You've got this! Keep learning and shining! - Ruchi Math Coaching