Finding Rational Numbers Between Two Rational Numbers

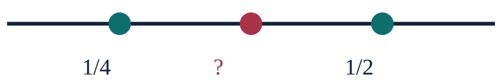
Unit: Rational Numbers - Student Practice Worksheet

Overview: Arjun said there was exactly one rational number between 1/4 and 1/2: namely 3/8, the midpoint. Meera asked him to find a SECOND one. He found it easily — and then realized his 'exactly one' claim was never going to survive.

Practice Questions (30 Total)

Section A - Easy

Q1. Number line from 1/4 to 1/2. Midpoint?



- [ ] **A.** 1/3
- [ ] **B.** 3/8
- [ ] **C.** 3/4
- [ ] **D.** 1/8

Q2. How many rational numbers exist between any two distinct rational numbers?

- [ ] **A.** Infinitely many
- [ ] **B.** Exactly one
- [ ] **C.** Exactly ten
- [ ] **D.** None

Q3. What formula gives a rational number exactly between a and b?

- [ ] **A.** a×b
- [ ] **B.** a–b
- [ ] **C.** (a+b)/2
- [ ] **D.** a/b

Q4. Rewrite 1/4 and 1/2 with denominator 40.

$1/4 \to 10/40$ $1/2 \to 20/40$

- [ ] **A.** 4/40 and 2/40
- [ ] **B.** 1/40 and 2/40
- [ ] **C.** 40/4 and 40/2
- [ ] **D.** 10/40 and 20/40

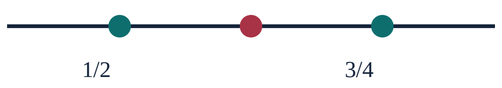
Q5. What term describes the property that rational numbers have infinitely many numbers between any two of them?

- [ ] **A.** Closure
- [ ] **B.** Commutativity
- [ ] **C.** Associativity
- [ ] **D.** Density

Q6. Is 15/40 a rational number strictly between 10/40 and 20/40?

- [ ] **A.** Yes
- [ ] **B.** No, it's outside the range
- [ ] **C.** No, it equals 10/40
- [ ] **D.** No, it equals 20/40

Q7. Number line from 1/2 to 3/4. Midpoint?



- [ ] **A.** 1/4
- [ ] **B.** 5/8
- [ ] **C.** 3/8
- [ ] **D.** 7/8

Q8. Can you find a rational number between two rational numbers that are very close together, like 1/100 and 2/100?

- [ ] **A.** No, they're too close
- [ ] **B.** Only if they're both whole numbers

- [ ] **C.** Yes, always possible
- [ ] **D.** Only if the denominators match exactly

Q9. Do integers have the same 'infinitely many numbers between any two' property as rational numbers?

- [ ] **A.** Yes, always
- [ ] **B.** No — integers like 3, 4 have gaps
- [ ] **C.** Only for negative integers
- [ ] **D.** Only for even integers

Q10. Midpoint of 2/5 and 3/5, simplified, and why it sits between them.

- [ ] **A.** 1/2
- [ ] **B.** 5/10
- [ ] **C.** 1/5
- [ ] **D.** 5/5

Section B - Medium

Q11. Between 10/40 and 20/40, pick three listed rationals.

$10/40 \quad \ldots \quad 20/40$

- [ ] **A.** 11/40, 15/40, 19/40
- [ ] **B.** 9/40, 21/40, 25/40
- [ ] **C.** 10/40, 20/40, 30/40
- [ ] **D.** 1/40, 2/40, 3/40

Q12. How many rational numbers strictly lie between 5/12 and 7/12?

- [ ] **A.** Exactly one: 6/12
- [ ] **B.** Infinitely many
- [ ] **C.** None
- [ ] **D.** Exactly two

Q13. To find 9 rational numbers between 1/4 and 1/2, what common denominator did the story use?

- [ ] **A.** 20

- [ ] **B.** 40
- [ ] **C.** 10
- [ ] **D.** 4

Q14. Midpoint of 3/8 and 1/2.

$$3/8 \quad \text{and} \quad 1/2$$

- [ ] **A.** 5/8
- [ ] **B.** 1/4
- [ ] **C.** 9/16
- [ ] **D.** 7/16

Q15. Is -1/2 a rational number between -1 and 0?

- [ ] **A.** No, outside the range
- [ ] **B.** No, negatives don't count
- [ ] **C.** Only as a decimal
- [ ] **D.** Yes, it lies between them

Q16. To find at least 15 rational numbers between 1/4 and 1/2, what scaling factor did the story use on the denominators?

- [ ] **A.** 10
- [ ] **B.** 8
- [ ] **C.** 16
- [ ] **D.** 15

Q17. Write 2/3 and 3/4 with denominator 24.

$$2/3 \rightarrow ?/24 \qquad 3/4 \rightarrow ?/24$$

- [ ] **A.** 16/24 and 18/24
- [ ] **B.** 8/24 and 6/24
- [ ] **C.** 2/24 and 3/24
- [ ] **D.** 24/2 and 24/3

Q18. Find 4 rational numbers between 1/5 and 2/5 using a common denominator of 25.

- ☐ **A.** 1/25, 2/25, 3/25, 4/25
- ☐ **B.** 11/25, 12/25, 13/25, 14/25
- ☐ **C.** 6/25, 7/25, 8/25, 9/25
- ☐ **D.** 5/25, 6/25, 7/25, 8/25

Q19. Using the denominators from M5 (16/24 and 18/24), find a rational number strictly between 2/3 and 3/4.

- ☐ **A.** 15/24
- ☐ **B.** 17/24
- ☐ **C.** 19/24
- ☐ **D.** 16/24

Q20. Midpoint of –1/3 and 1/3, and why 0 is still rational.

- ☐ **A.** 0
- ☐ **B.** 1/3
- ☐ **C.** -1/3
- ☐ **D.** 2/3

Section C - Challenge

Q21. 1/6 and 1/3 as 48ths. Which listing gives 7 rationals between them?

$1/6 = 8/48$ $1/3 = 16/48$

- ☐ **A.** 48ths: 9/48 through 15/48
- ☐ **B.** 18ths: 1/18, 2/18 only
- ☐ **C.** 18ths: 7/18 through 13/18
- ☐ **D.** No denominator works

Q22. Between 0 and 1/1000, does the density property of rational numbers still guarantee infinitely many rational numbers?

- ☐ **A.** No, intervals below 1/100 are exceptions
- ☐ **B.** No, only intervals containing whole numbers qualify
- ☐ **C.** Yes, regardless of how small the interval is

- [ ] **D.** Only if 0 is excluded from consideration

Q23. 1/2 and 2/3 are already at a common denominator of 6 (3/6 and 4/6). Scaling by 6 gives 18/36 and 24/36. How many whole numerators lie strictly between 18 and 24?

- [ ] **A.** None at all
- [ ] **B.** 5 numerators
- [ ] **C.** 6 numerators
- [ ] **D.** 10 numerators

Q24. Between 42/54 and 48/54, pick a rational that is not the midpoint.

$7/9 = 42/54$ $8/9 = 48/54$

- [ ] **A.** 46/54
- [ ] **B.** 7.5/9
- [ ] **C.** 15/18, the midpoint
- [ ] **D.** 1, an endpoint

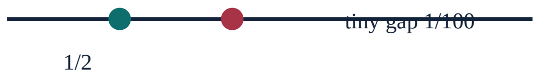
Q25. Are there any TWO rational numbers between which NO other rational number can be found?

- [ ] **A.** Yes, if both are whole numbers
- [ ] **B.** Yes, if both are negative
- [ ] **C.** Yes, if the difference is 1
- [ ] **D.** No, density rules this out

Q26. Between 3/7 and 4/7, using a common denominator of 70, how many whole-number numerators lie strictly between the two scaled numerators?

- [ ] **A.** 10
- [ ] **B.** 9
- [ ] **C.** 7
- [ ] **D.** 70

Q27. Tiny gap after 1/2. How large must the numerator gap be to fit one rational?



- [ ] **A.** Gap must be at least 2
- [ ] **B.** No scaling ever works
- [ ] **C.** Gap must be exactly 2
- [ ] **D.** Gap must equal 100

Q28. To guarantee at least 20 rational numbers between two fractions currently at a common denominator with numerators differing by only 3, what must you do?

- [ ] **A.** Nothing, 3 is already enough
- [ ] **B.** Scale the denominator DOWN
- [ ] **C.** Scale the denominator up so the numerator gap becomes at least 21
- [ ] **D.** Switch to decimals instead

Q29. A student claims that repeatedly finding midpoints will eventually 'run out' of new rational numbers between 1/4 and 1/2. Is this claim correct?

- [ ] **A.** Yes, after exactly 10 steps
- [ ] **B.** Yes, after exactly 100 steps
- [ ] **C.** Yes, once the fractions become too small to write
- [ ] **D.** No — each new midpoint always creates fresh room on both sides

Q30. Does density mean the number line has no gaps at all, or do irrationals still sit in the reals?

- [ ] **A.** Yes, completely correct
- [ ] **B.** Not quite — irrationals like √2 fill real gaps
- [ ] **C.** No, rationals have large gaps
- [ ] **D.** No, density needs whole numbers

Answer Key

Q1: (B) (b) 3/8. (1/4+1/2)/2=(3/4)/2=3/8.

Q2: (A) (a) Infinitely many. This density property is a key feature of rational numbers.

Q3: (C) (c) (a+b)/2. The average of two numbers always lies exactly between them.

Q4: (D) (d) 10/40 and 20/40. Multiply numerator and denominator of each by the appropriate factor.

Q5: (D) (d) Density. This is the standard term for this property of rational numbers.

Q6: (A) (a) Yes. 15 lies strictly between 10 and 20, so 15/40 lies strictly between 10/40 and 20/40.

Q7: (B) (b) 5/8. (1/2+3/4)/2=(5/4)/2=5/8.

Q8: (C) (c) Yes, always possible. The density property holds no matter how close two distinct rational numbers are.

Q9: (B) (b) No — consecutive integers like 3 and 4 have no integer between them. This is exactly what makes rational numbers special: they're dense, integers are not.

Q10: (A) (a) 1/2. (2/5+3/5)/2=(5/5)/2=1/2 — note 5/10 also equals 1/2, so both describe the same value, but 1/2 is the simplest form.

Q11: (A) (a) 11/40, 15/40, 19/40. Each numerator lies strictly between 10 and 20.

Q12: (B) (b) Infinitely many. Even though 6/12 is the only one at THAT particular denominator, scaling to a bigger denominator reveals infinitely more.

Q13: (B) (b) 40. This gave numerators 10 and 20, with 9 whole numbers strictly in between (11 through 19).

Q14: (D) (d) 7/16. (3/8+1/2)/2=(3/8+4/8)/2=(7/8)/2=7/16.

Q15: (D) (d) Yes, it lies between them. -1/2 lies exactly halfway between -1 and 0, showing density applies to negative rationals too.

Q16: (C) (c) 16. Scaling by 16 gave denominators of 64 with numerators 16 and 32, leaving 15 numbers strictly in between.

Q17: (A) (a) 16/24 and 18/24. 2/3×8/8=16/24, and 3/4×6/6=18/24.

Q18: (C) (c) 6/25, 7/25, 8/25, 9/25. 1/5=5/25 and 2/5=10/25, so numerators 6 through 9 lie strictly between them.

Q19: (B) (b) 17/24. 17 lies strictly between 16 and 18, the only whole number available at this denominator.

Q20: (A) (a) 0. (-1/3+1/3)/2=0/2=0.

Q21: (A) (a) 48ths: 9/48 through 15/48. 1/6=8/48 and 1/3=16/48, giving 7 whole numerators (9 to 15) strictly in between.

Q22: (C) (c) Yes, regardless of how small the interval is. The density property has no lower bound on interval size — it holds for any two distinct rational numbers, however close.

Q23: (B) (b) 5 numerators. The gap of 6 (24–18) leaves exactly 5 whole numbers (19,20,21,22,23) strictly inside it.

Q24: (A) (a) 46/54. Scaling 7/9=42/54 and 8/9=48/54 gives several genuine in-between values like 46/54, distinct from the midpoint 15/18.

Q25: (D) (d) No, density rules this out. The density property is universal across all pairs of distinct rational numbers, without exception.

Q26: (B) (b) 9. 3/7=30/70 and 4/7=40/70; numerators 31 through 39 are strictly in between, giving 9 values.

Q27: (A) (a) Gap must be at least 2. Any sufficiently large common denominator creates the needed gap — the exact multiplier isn't fixed, only that it must be large enough.

Q28: (C) (c) Scale the denominator up so the numerator gap becomes at least 21. A numerator gap of g leaves g–1 whole numbers strictly in between, so you need $g \geq 21$ for at least 20 numbers.

Q29: (D) (d) No — each new midpoint always creates fresh room on both sides. The midpoint process can be repeated indefinitely, generating infinitely many distinct rational numbers.

Q30: (B) (b) Not quite — irrationals like $\sqrt{2}$ fill real gaps. Density (infinitely many rationals in any interval) is a weaker property than completeness (no gaps at all) — the real number line needs irrational numbers too.

You've got this! Keep learning and shining! - Ruchi Math Coaching